Statistics

Lecture 11

Feb 19-8:47 AM

SG 20

Store 2,4,6 in L1

use [1-Var Stats]

with L1 only to find

$\mu = \bar{x} = 4$

$\sigma = 1.633$

$\sigma^2 = \frac{8}{3}$

Take all Samples of Size 2 with replacement.

2,2	2,4	2,6
4,2	4,4	4,6
6,2	6,4	6,6

Find $\bar{x}$ of each Sample

2	3	4
3	4	5
4	5	6

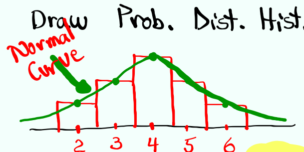
9 Means

$\bar{x}$	$P(\bar{x})$
2	1/9
3	2/9
4	3/9
5	2/9
6	1/9

$\bar{x} \to L2$

$P(\bar{x}) \to L3$

Draw Prob. Dist. Hist.



use [1-Var Stats] with L2 & L3 to find

$\mu = 4$

$\sigma = 1.155$

$\sigma^2 = \frac{4}{3} = \frac{8/3}{2}$

Nov 8-8:07 AM

Central - Limit Theorem

CLT

$\mu_{\bar{x}} = \mu$ $\quad \sigma_{\bar{x}}^2 = \frac{\sigma^2}{n}$ $\quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

Ages of college students are normally dist. with mean of 30 Yrs and standard dev. of 5 Yrs.

If we take all samples of Size 4 from this population,

CLT
$$\mu_{\bar{x}} = \mu = 30$$
$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{5}{\sqrt{4}} = \frac{5}{2} = 2.5$$

Nov 8-8:20 AM

Clear all lists

Store 2, 4, 6, and 8 in L1.

use 1-Var Stats with L1 only to find

$\mu = 5$

$\sigma = 2.236$

$\sigma^2 = 5$

Let's take all Samples with Size 2 with replacement.

2,2	2,4	2,6	2,8
4,2	4,4	4,6	4,8
6,2	6,4	6,6	6,8
8,2	8,4	8,6	8,8

Find $\bar{x}$ of each Sample

2	3	4	5
3	4	5	6
4	5	6	7
5	6	7	8

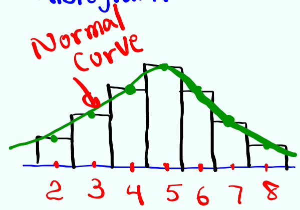
Nov 8-8:26 AM

2	3	4	5
3	4	5	6
4	5	6	7
5	6	7	8

16 means

$\bar{x}$	$P(\bar{x})$
2	1/16
3	2/16
4	3/16
5	4/16
6	3/16
7	2/16
8	1/16

Draw Prob. dist. histogram:



$\mu_{\bar{x}} = 5$

$\sigma_{\bar{x}} = 1.581$

$\sigma^2_{\bar{x}} = \frac{5}{2}$

$\bar{x} \to L2$

$P(\bar{x}) \to L3$

Use 1-Var Stats with L2 & L3 to find

CLT

$\mu_{\bar{x}} = \mu$ ✓

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$ ✓

$\sigma^2_{\bar{x}} = \frac{\sigma^2}{n}$ ✓

Given $N(75, 10)$

For randomly selected groups of 16,

$\mu_{\bar{x}} = \mu = 75$

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{16}} = \frac{10}{4} = 2.5$

Nov 8-8:32 AM

Clear all lists

Store 2, 4, 6, 8, and 10 in L1.

Use 1-Var Stats with L1 only to find

$\mu = 6$

$\sigma = 2.828$

$\sigma^2 = 8$

Take all Samples of Size 2 with replacement

2,2 2,4 2,6 2,8 2,10

4,2 4,4 - - - - - - -

6,2 6,4 - - - - -

⋮

10,2 10,4 - - - - - 10,10

Find $\bar{x}$ of each Sample

2	3	4	5	6
3	4	5	6	7
4	5	6	7	8
5	6	7	8	9
6	7	8	9	10

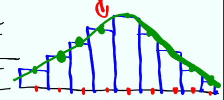
Nov 8-8:43 AM

2	3	4	5	6
3	4	5	6	7
4	5	6	7	8
5	6	7	8	9
6	7	8	9	10

25 means

$\bar{x}$	$P(\bar{x})$
2	1/25
3	2/25
4	3/25
5	4/25
6	5/25
7	4/25
8	3/25
9	2/25
10	1/25

Normal Curve



$\mu_{\bar{x}} = 6$

$\sigma_{\bar{x}} = 2$

$\sigma^2_{\bar{x}} = 4 = \frac{8}{2}$

$\bar{x} \to$ L2

$P(\bar{x}) \to$ L3

Use 1-Var Stats with L2 & L3 to find

Salaries of teachers are N.D. with $\mu = \$7800$ and $\sigma = 400$. If we randomly select all samples of size 25, find

$\mu_{\bar{x}} = \mu = 7800$

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{400}{\sqrt{25}} = \frac{400}{5} = 80$

CLT

SG 20 ✓

Nov 8-8:48 AM

SG 21

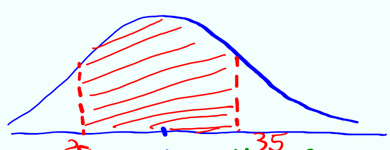
Suppose ages of College students are normally dist. with mean of 32 Yrs and Standard dev. of 6 Yrs. $N(32, 6)$

If we take samples of size 4 students, ($n = 4$) find the Prob. that their mean age ($\bar{x}$) is between 28 an 35 Yrs.

$P(28 < \bar{x} < 35)$

$= \text{normalcdf}(28, 35, 32, 3)$

$= \boxed{.750}$

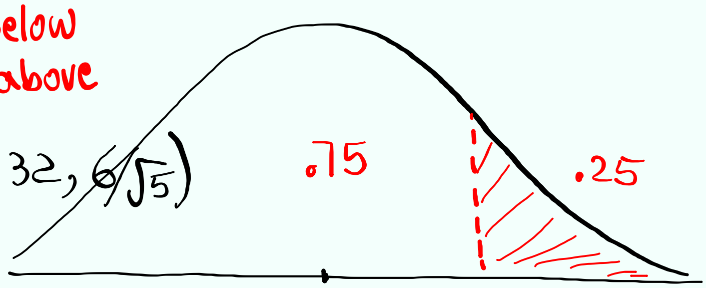


CLT $\begin{cases} \mu_{\bar{x}} = \mu = 32 \\ \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{6}{\sqrt{4}} = 3 \end{cases}$

Nov 8-9:11 AM

For randomly Selected groups of 5 students, find $\bar{x} = Q_3$, Round to whole #.

75% below
25% above



$\bar{x}$ = invNorm(.75, 32, 6/$\sqrt{5}$)

$\approx$ **34** Yrs

CLT $\begin{cases} \mu_{\bar{x}} = \mu = 32 \\ \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{6}{\sqrt{5}} \end{cases}$ $\bar{x} = Q_3$

Nov 8-9:18 AM

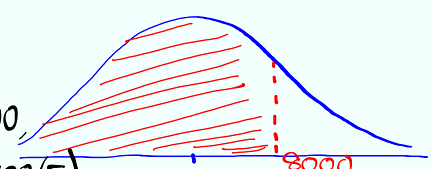
Salaries of teachers are normally dist. with the mean of \$7800 and standard dev. of \$400. N(7800, 400)

If we randomly Select ($\bar{x}$) 6 teachers ($n = 6$) find the Prob. that their mean Salary is below \$8000.

$P(\bar{x} < 8000)$

= normalcdf(-E99, 8000, 7800, 400/$\sqrt{6}$)

$\approx$ **.890**



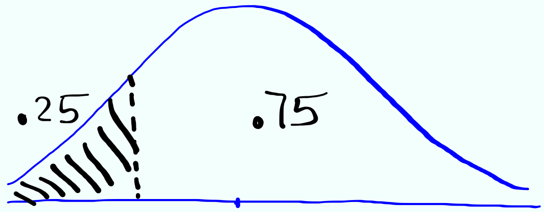
CLT $\begin{cases} \mu_{\bar{x}} = \mu = 7800 \\ \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{400}{\sqrt{6}} \end{cases}$

Nov 8-9:23 AM

For randomly selected group of 10 teachers, find $\bar{x} = Q_1$. Round to whole #.

25% below

75% above



$\bar{x} = \text{invNorm}(.25, 7800, 400/\sqrt{10})$

CLT $\begin{cases} \mu_{\bar{x}} = \mu = 7800 \\ \sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{400}{\sqrt{10}} \end{cases}$

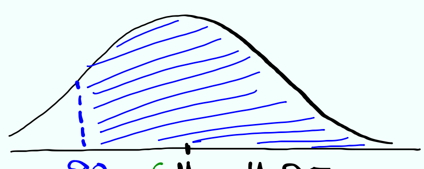
$\approx \$7715$

Nov 8-9:29 AM

Exam Scores are normally dist. with mean of 85 and standard dev. of 8.

If we randomly select 3 exams ($n = 3$), find the prob. that their mean score ($\bar{x}$) is above 80.

$P(\bar{x} > 80)$



CLT $\begin{cases} \mu_{\bar{x}} = \mu = 85 \\ \sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{8}{\sqrt{3}} \end{cases}$

$= \text{normalcdf}(80, E99, 85, 8/\sqrt{3})$

$\approx$ **.860**

Nov 8-9:34 AM

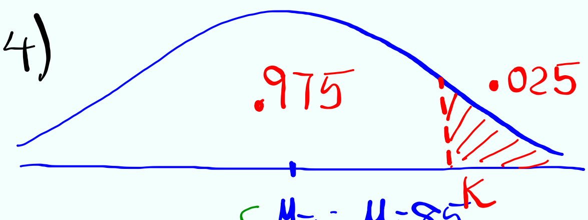
Find K such that $P(\bar{x} > K) = .025$ (Right Tail Area)

for randomly selected 4 exams.

Round to a whole #.

$K = \text{invNorm}(.975, 85, 4)$

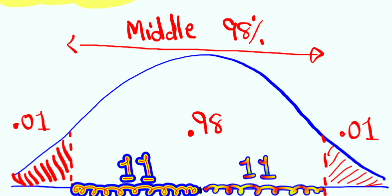
$\approx \boxed{93}$



CLT $\begin{cases} \mu_{\bar{x}} = \mu = 85 \\ \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{8}{\sqrt{4}} = 4 \end{cases}$

Nov 8-9:39 AM

Find two means, round to whole numbers, for randomly selected 3 exams that Separate the middle 98% from the rest.



$\bar{x}_1 = 74$ $\quad$ $\bar{x}_2 = 96$

CLT $\begin{cases} \mu_{\bar{x}} = \mu = 85 \\ \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{8}{\sqrt{3}} \end{cases}$

$\bar{x}_1 = \text{invNorm}(.01, 85, 8/\sqrt{3})$

$\approx \boxed{74}$

$\bar{x}_2 = \text{invNorm}(.99, 85, 8/\sqrt{3})$

$\approx \boxed{96}$

SG 21

Exam 3: SG 1 – SG 23

Nov. 22 Two weeks from Today

Nov 8-9:44 AM

α Alpha

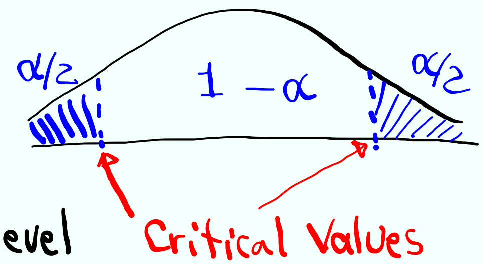
$0 < \alpha < 1$

α Significance level

$1-\alpha$ Middle Area

$(1-\alpha)\cdot 100\%$ Confidence level

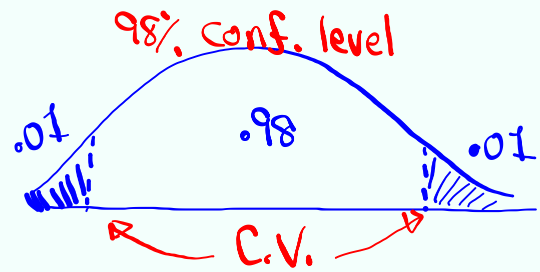
$\alpha/2$ is the area on each tail.



ex: $\alpha = .02$

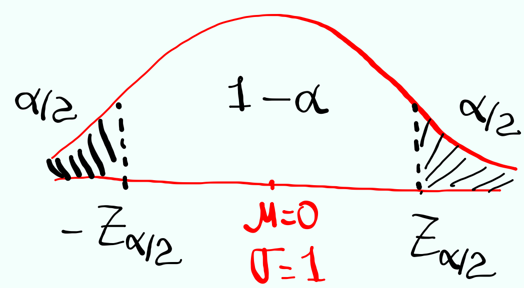
$\alpha/2 = .01$

$1-\alpha = .98$



Nov 8-10:12 AM

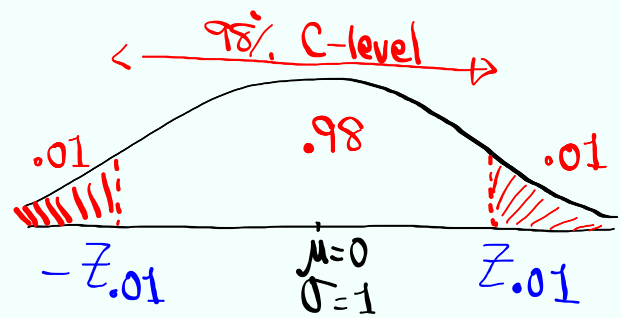
$Z_{\alpha/2}$ is the Critical Value with right tail area of $\alpha/2$.



Find $Z_{.01}$

$Z_{.01} = \text{invNorm}(.99, 0, 1)$

$= \boxed{2.326}$



Nov 8-10:16 AM

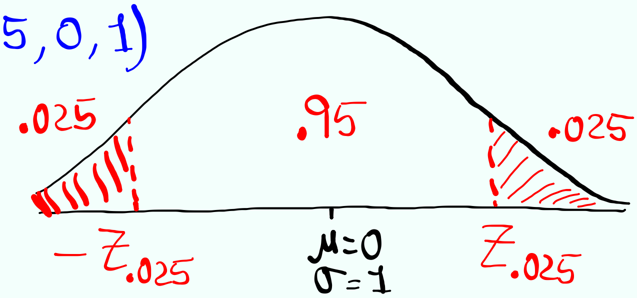
Find $\pm Z_{\alpha/2}$ for $\alpha = .05$.

$\alpha/2 = .025$ Area of each tail

$1 - \alpha = .95$ Middle Area

$Z_{.025} = \text{invNorm}(.975, 0, 1)$

$= \boxed{1.960}$



Nov 8-10:21 AM

Find $\pm Z_{\alpha/2}$ for 90% Conf. level.

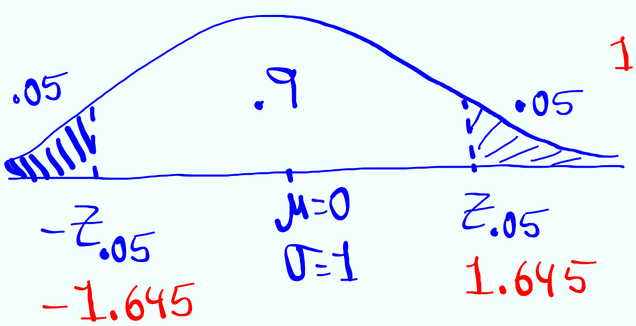
Middle Area

.9

$1 - \alpha = .9$

$\alpha = .1$

$\alpha/2 = .05$



$Z_{.05} = \text{invNorm}(.95, 0, 1) = \boxed{1.645}$

Nov 8-10:25 AM

t - Dist.:

It is very Similar to Z-Dist.

1) Symmetric, Bell-Shape, total Area = 1

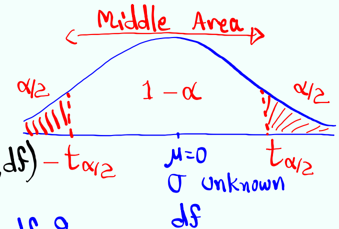
2) $\mu = 0$, σ unknown

3) It comes with degrees of freedom.

How to find $t_{\alpha/2}$:

[2nd] [VARS]

invT(Left Area, df) — $t_{\alpha/2}$

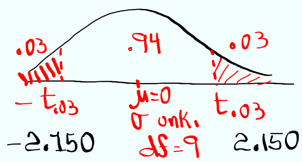


find $t_{.03}$ with df = 9.

$\alpha/2 = .03$

$\alpha = .06$

$1 - \alpha = .94$



invT(.97, 9) = 2.150

Nov 8-10:28 AM

find $\pm t_{\alpha/2}$ for 98% Conf. level with df = 19.

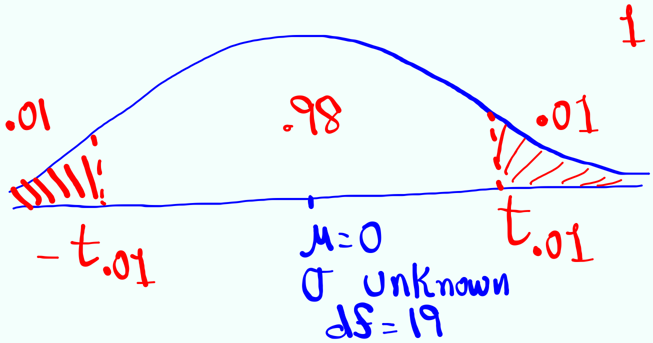
Middle Area

.98

$1 - \alpha = .98$

$\alpha = .02$

$\alpha/2 = .01$



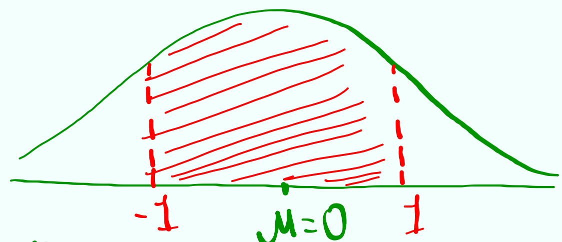
invT(.99, 19) ≈ 2.539

Nov 8-10:35 AM

Find $P(-1 < t < 1)$ with $df = 999$.

use

tcdf(L, U, df)



tcdf(-1, 1, 999) = **.682**

σ unknown
df = 999

≈ 68%

As df increases, t-dist ≈ Z-dist.

Nov 8-10:39 AM

What is degrees of freedom?

We find df differently by topics.

12 Students

I bring 12 donuts, You can choose one donut to eat.

First Student → 12 donuts → 12 choices
Second " → 11 " → 11 choices
⋮
Last " → 1 donut → 0 choice

df = 11

You have 7 clean Shirts.

You wear a clean shirt everyday.

Monday → 7 choices
Tuesday → 6 "
Wednesday → 5 "
⋮
Sun → No choice (only one clean shirt)

df = 6

Do SG 20 & SG 21.

Look for my Canvas email for extra Credit Study Guide.

Nov 8-10:43 AM